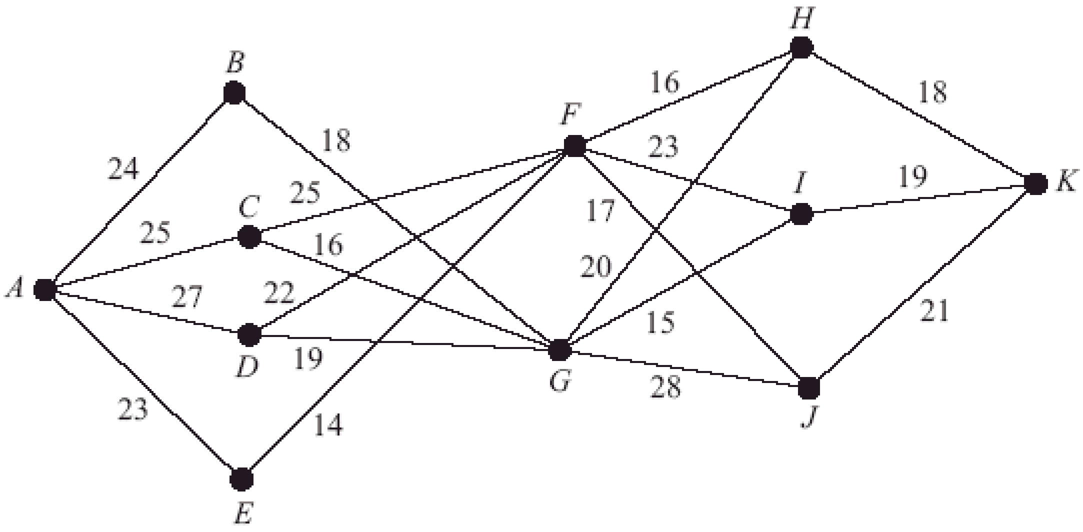
1.



A maximin route is to be found through the network shown in the figure above.

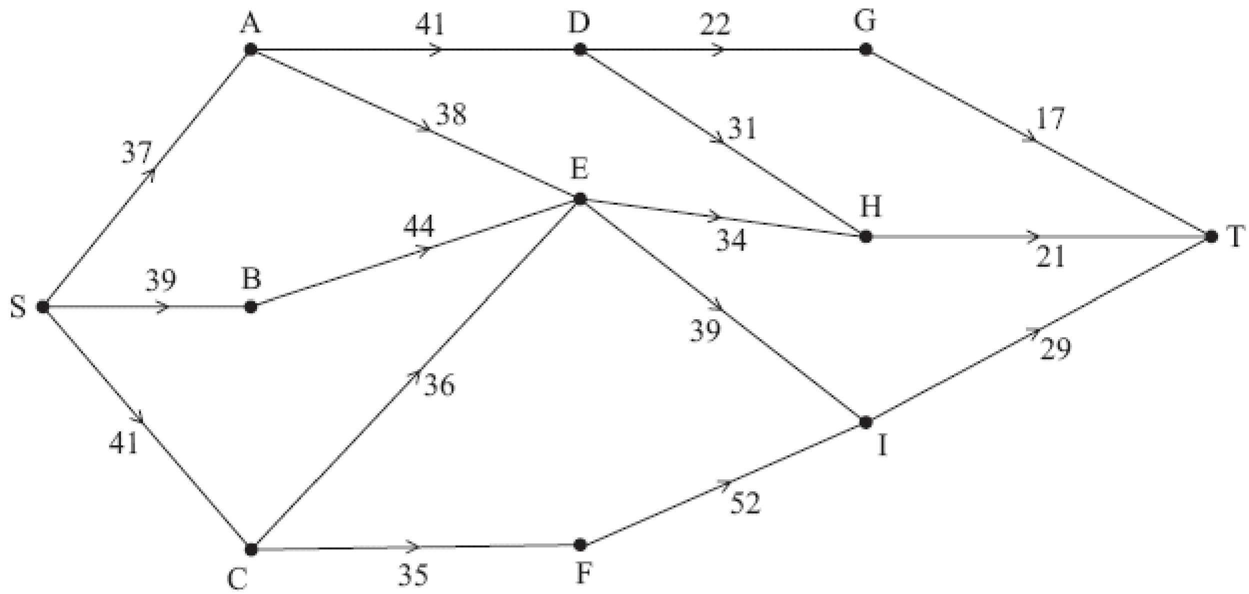
Complete the table below, and hence find a maximin route.

You may not wish to use all of these lines.

Stage	State	Action	Value

(Total 9 marks)

2.



The diagram above represents the maintenance choices a council can make and their costs, in £1000s, over the next four years.

The council wishes to minimise the greatest annual cost of maintenance.

(a) Use dynamic programming to find a minimax route from S to T.

You may need to use all these rows.

Stage	State	Action	Destination	Value

Stage	State	Action	Destination	Value

(9)

(b) State your route and the greatest annual cost incurred by the council.

(2)

(c) Calculate the average annual cost to the council.

(2)
(Total 13 marks)

3. Minty has £250 000 to allocate to three investment schemes. She will allocate the money to these schemes in units of £50 000. The net income generated by each scheme, in £1000s, is given in the table below.

	£0	**£50 000**	**£100 000**	**£150 000**	**£200 000**	**£250 000**
Scheme 1	0	60	120	180	240	300
Scheme 2	0	65	125	190	235	280
Scheme 3	0	55	110	170	230	300

Minty wishes to maximise the net income. She decides to use dynamic programming to determine the optimal allocation, and starts the table shown in your answer book.

(a) Complete the table below to determine the amount Minty should allocate to each scheme in order to maximise the income. State the maximum income and the amount that should be allocated to each scheme.

Stage	**State (in £1000s)**	**Action (in £1000s)**	**Destination (in £1000s)**	**Value (in £1000s)**
1	250	250	0	300 *
	200	200	0	240 *
	150	150	0	180 *
	100	100	0	120 *
	50	50	0	60 *
	0	0	0	0 *
2	250	250	0	280 + 0 = 280
		200	50	235 + 60 = 295
		150	100	
		100	150	
		50	200	
		0	250	
	200	200	0	

		150 100 50 0	50	
	150	150 100 50 0		
	100			

Stage	State (in £1000s)	Action (in £1000s)	Destination (in £1000s)	Value (in £1000s)

(10)

(b) For this problem give the meaning of the table headings

(i) Stage,

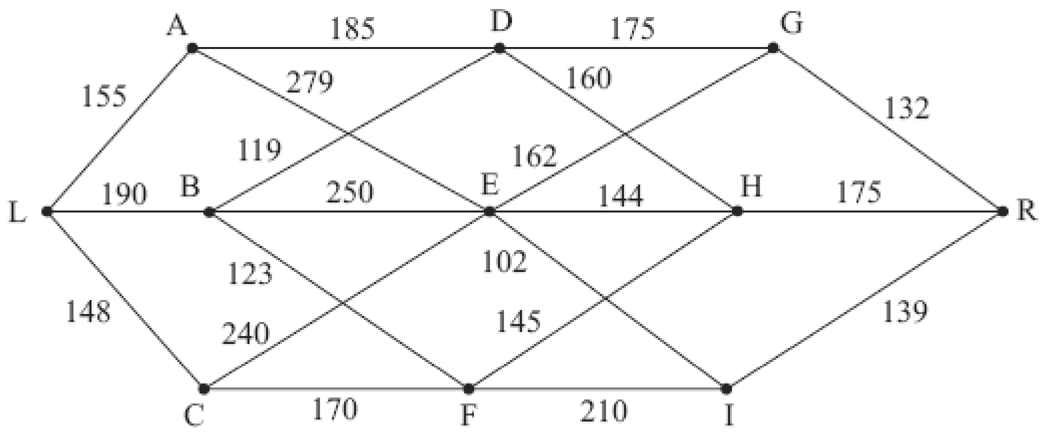
(ii) State,

(iii) Action.

(3)
(Total 13 marks)

4. (a) Explain the difference between a maximin route and a minimax route in dynamic programming.

(2)



A maximin route from L to R is to be found through the staged network shown above.

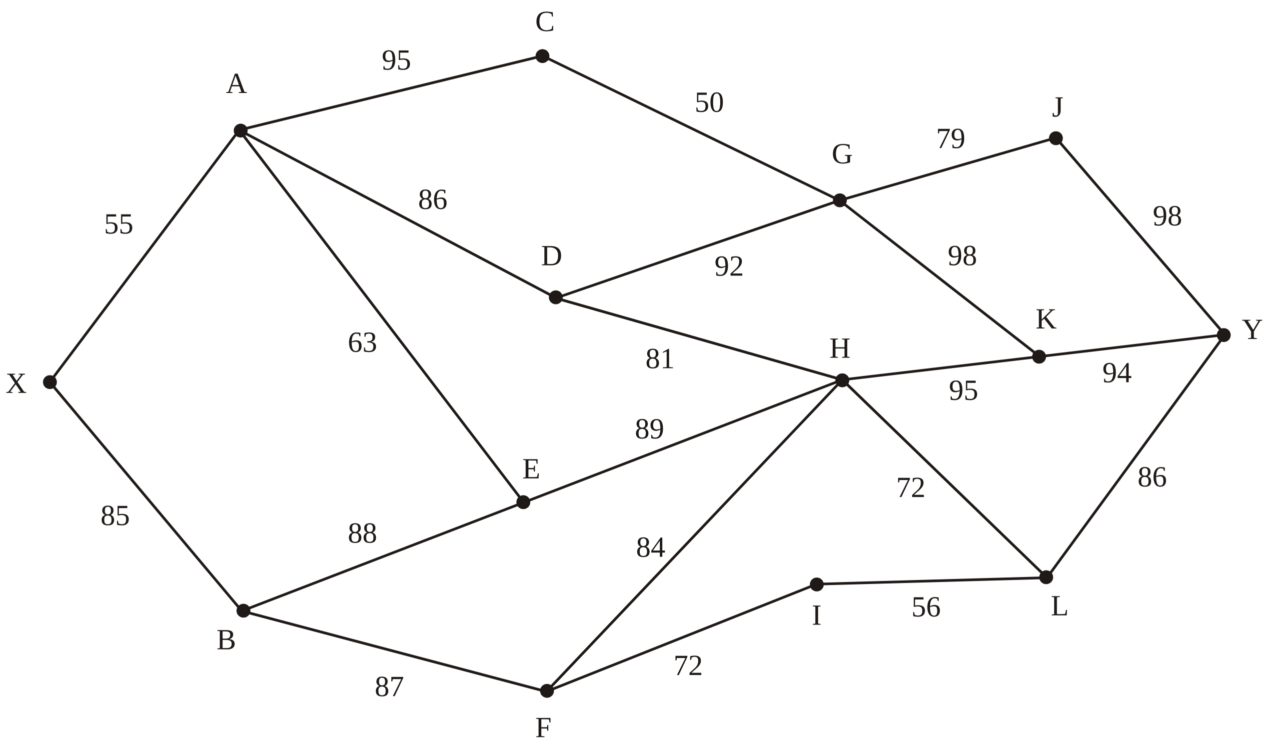
(b) Use dynamic programming to complete the table below and hence find a maximin route.

Stage	State	Action	Destination	Value

Stage	State	Action	Destination	Value

(10)
(Total 12 marks)

5.



Agent Goodie has successfully recovered the stolen plans from Evil Doctor Fiendish and needs to take them from Evil Doctor Fiendish's secret headquarters at X to safety at Y. To do this he must swim through a network of underwater tunnels. Agent Goodie has no breathing apparatus, but knows that there are twelve points, *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H*, *I*, *J*, *K* and *L*, at which there are air pockets where he can take a breath.

The network is modelled above, and the number on each arc gives the time, in seconds, it takes Agent Goodie to swim from one air pocket to the next.

Agent Goodie needs to find a route through this network that minimises the longest time between successive air pockets.

(a) Use dynamic programming to complete the table below and hence find a suitable route for Agent Goodie.

Stage	State	Action	Destination	Value

Stage	State	Action	Destination	Value

(12)

Unfortunately, just as Agent Goodie is about to start his journey, tunnel XA becomes blocked.

(b) Find an optimal route for Agent Goodie avoiding tunnel XA.

(2)
(Total 14 marks)

6. (a) State Bellman's principle of optimality.

(1)

(b) Explain what is meant by a minimax route.

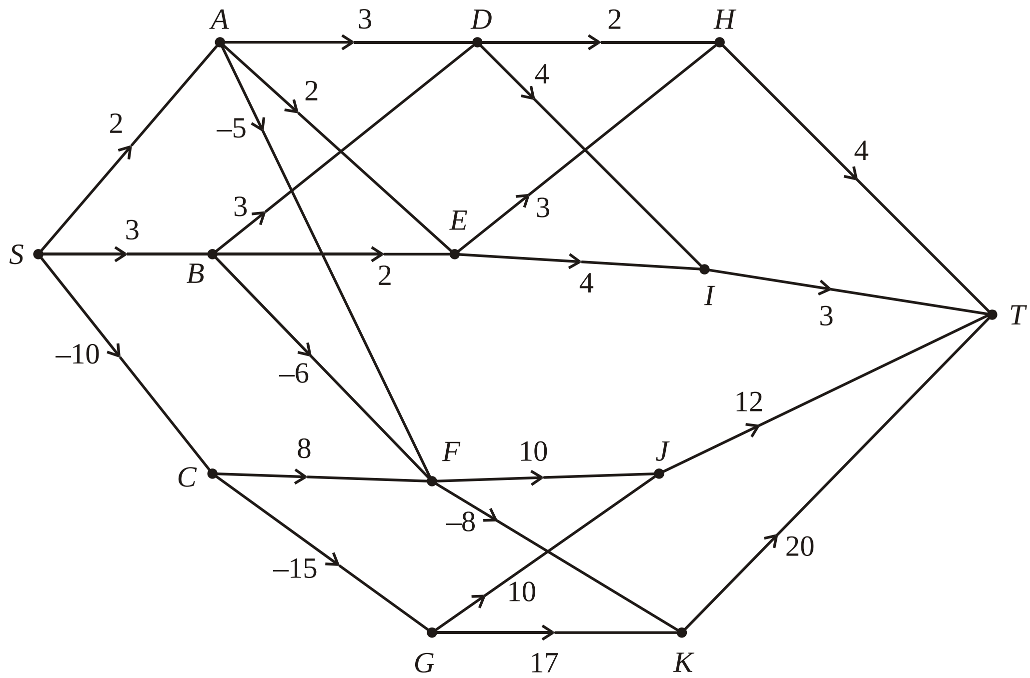
(1)

(c) Describe a practical problem that would require a minimax route as its solution.

(2)
(Total 4 marks)

7. Victor owns some kiosks selling ice cream, hot dogs and soft drinks.

The network below shows the choices of action and the profits, in thousands of pounds, they generate over the next four years. The negative numbers indicate losses due to the purchases of new kiosks.



Use a suitable algorithm to determine the sequence of actions so that the profit over the four years is maximised and state this maximum profit.

(Total 12 marks)

8. An engineering firm makes motors. They can make up to five in any one month, but if they make more than four they have to hire additional premises at a cost of £500 per month. They can store up to two motors for £100 per motor per month. The overhead costs are £200 in any month in which work is done.
Motors are delivered to buyers at the end of each month. There are no motors in stock at the beginning of May and there should be none in stock after the September delivery.

The order book for motors is:

Month	May	June	July	August	September
Number of motors	3	3	7	5	4

Use dynamic programming to determine the production schedule that minimises the costs, showing your working in the table provided below.

Stage (month)	State (Number in store at start of month)	Action (Number made in month)	Destination (Number in store at end of month)	Value (cost)

Production schedule

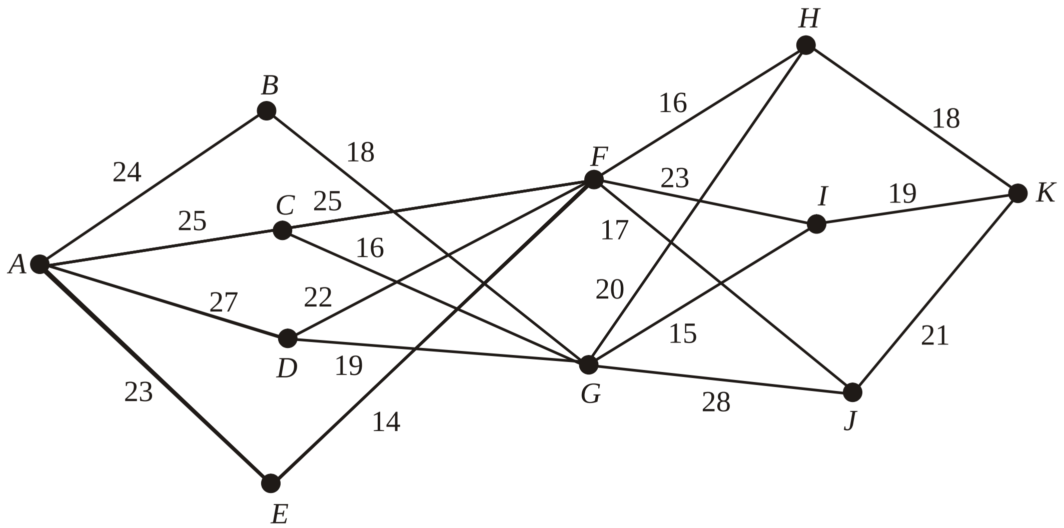
Month	May	June	July	August	September
Number to be made					

Total cost: ....

(Total 12 marks)

9. (a) Explain what is meant by a maximin route in dynamic programming, and give an example of a situation that would require a maximin solution.

(3)



A maximin route is to be found through the network shown in the diagram.

(b) Complete the table in the answer book, and hence find a maximin route.

(9)

(c) List **all** other maximin routes through the network.

(2)
(Total 14 marks)

10. Joan sells ice cream. She needs to decide which three shows to visit over a three-week period in the summer. She starts the three-week period at home and finishes at home. She will spend one week at each of the three shows she chooses travelling directly from one show to the next.

Table 1 gives the week in which each show is held. Table 2 gives the expected profits from visiting each show. Table 3 gives the cost of travel between shows.

Table 1

Week	1	2	3
Shows	A, B, C	D, E	F, G, H

Table 2

Show	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Expected Profit (£)	900	800	1000	1500	1300	500	700	600

Table 3

Travel costs (£)	A	B	C	D	E	F	G	H
Home	70	80	150			80	90	70
A				180	150			
B				140	120			
C				200	210			
D						200	160	120
E						170	100	110

It is decided to use dynamic programming to find a schedule that maximises the total expected profit, taking into account the travel costs.

(a) Define suitable stage, state and action variables.

(3)

(b) Determine the schedule that maximises the total profit. Show your working in the table below.

Stage	State	Action	Value

Stage	State	Action	Value

(12)

(c) Advise Joan on the shows that she should visit and state her total expected profit.

(3)
(Total 18 marks)

11. Kris produces custom made racing cycles. She can produce up to four cycles each month, but if she wishes to produce more than three in any one month she has to hire additional help at a cost of £350 for that month. In any month when cycles are produced, the overhead costs are £200. A maximum of 3 cycles can be held in stock in any one month, at a cost of £40 per cycle per month. Cycles must be delivered at the end of the month. The order book for cycles is

Month	August	September	October	November
Number of cycles required	3	3	5	2

Disregarding the cost of parts and Kris' time,

(a) determine the total cost of storing 2 cycles and producing 4 cycles in a given month, making your calculations clear.

(2)

There is no stock at the beginning of August and Kris plans to have no stock after the November delivery.

(b) Use dynamic programming to determine the production schedule which minimises the costs, showing your working in the table below.

Stage	Demand	State	Action	Destination	Value
1 (Nov)	2	0 (in stock)	(make) 2	0	200
		1 (in stock)	(make) 1	0	240
		2 (in stock)	(make) 0	0	80
2 (Oct)	5	1	4	0	590 + 200 = 790
		2	3 4	0 1	
		3	2		

Production schedule

Month	August	September	October	November
Number of cycles to be made				

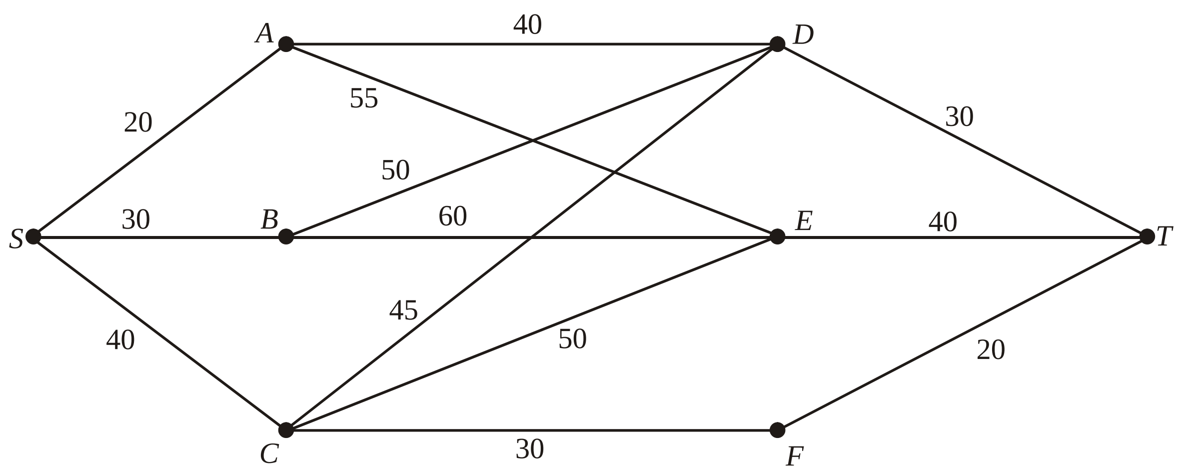
(13)

The fixed cost of parts is £600 per cycle and of Kris' time is £500 per month. She sells the cycles for £2000 each.

(c) Determine her total profit for the four month period.

(3)
(Total 18 marks)

12. Jenny wishes to travel from S to T. There are several routes available. She wishes to choose the route on which the maximum altitude, above sea level, is as small as possible. This is called the minimax route.



The diagram above gives the possible routes and the weights on the edges give the maximum altitude on the road (in units of 100 feet).

Use dynamic programming, carefully defining the stages and states, to determine the route or routes Jenny should take. You should show your calculations in tabular form, using a table with columns labelled as shown below.

Stage	Initial State	Action	Final State	Value

(Total 12 marks)

13. A builder owns three areas of land on which he wishes to build. The planning authority decides that each area of land must have a different type of building. Once the type of building has been chosen, every building in that area must be of the same design. The profit made by the builder depends on the area of land and the type of building.

	Area of Land		
Type of Building	**1**	**2**	**3**
Detached House (*D*)	60	56	50
Semi-detached House (*S*)	50	45	35
Bungalow (*B*)	60	50	45

The table shows the profits in units of £1000.

The builder wishes to *maximise* his overall profit. Use the Hungarian algorithm to decide which type of building should be allocated to which area of land. State the maximum profit.

(Total 13 marks)

1.

Stage	State	Action	Value	
1	*H*	*HK*	18 *	
	I	*IK*	19 *	M1 A1
	J	*JK*	21 *	
2	*F*	**FH**	min(16, 18) = 16	M1 A1 A1
		FI	min (23, 19) = 19 *	
		FJ	min(17, 21) = 17	
	G	*GH*	min(20, 18) = 18	
		GI	min(15, 19) = 15	A1
		GJ	min(28, 21) = 21 *	
3	*B*	*BG*	min(18, 21) = 18 *	M1 A1ft
	C	*CF*	min(25, 19) = 19 *	
		CG	min(16, 21) = 16	
	D	*DF*	min(22, 19) = 19 *	
		DG	min(19, 21) = 19 *	
	E	*EF*	min(14, 19) = 14 *	
4	*A*	*AB*	min(24, 18) = 18	A1ft
		AC	min(25, 19) = 19 *	
		AD	min(27, 19) = 19 *	
		AE	min(23, 14) = 14	

Routes A C F I K, or A D F I K or A D G J K A1 ft 9

[9]

2. (a) Minimax route

Stage	State	Action	Dest.	Value	
	G	GT	T	17 *	1M1
1	H	HT	T	21 *	A1
	I	IT	T	29 *	
2	D	DG	G	max(22, 17) = 22 *	2M1 A1
		DH	H	max(31, 21) = 31	
	E	EH	H	max(34, 21) = 34 *	A1
		EI	I	max(39, 29) = 39	
	F	FI	I	max(52, 29) = 52 *	
3	A	AD	D	max(41, 22) = 41	
		AE	E	max(38, 34) = 38 *	3M1 A1ft
	B	BE	E	max(44, 34) = 44 *	
	C	CE	E	max(36, 34) = 36 *	A1ft
		CF	F	max(35, 52) = 52	
4	S	SA	A	max(37, 38) = 38 *	
		SB	B	max(39, 44) = 44	A1

		SC	C	max(41, 36) = 41	9

Notes

Throughout section (a):

- **Condone lack of destination column and/or reversed stage numbers throughout.**
- **Only penalise incorrect result in Value – ie ignore working values.**
- **Penalise absence of state or action column with first two A marks earned only**
- **Penalise empty/errors in stage column with first A mark earned only.**

1M1: First, T, stage complete and working backwards.
1A1: CAO (condone lack of *)
2M1: Second stage completed. Penalise reversed states here and in (b). Bod if something in each column.
2A1: Any 2 states correct. Penalise * errors, with an A mark, only once in the question).
3A1: All 3 states correct. (Penalise * errors only once in the question).
3M1: 3^{rd} and 4^{th} stages completed. Bod if something in each column.
4A1ft: Any 2 states correct. (Penalise * errors only once in the question). A, B or C
5A1ft: All 3 states correct. (Penalise * errors only once in the question). A, B and C.
6A1ft: Final, S, state correct. (Penalise * errors only once in the question).

(b) Route: SAEHT Greatest annual cost: £38 000 M1 A1ft 2

Notes

1M1: Route (S to T or vv.) and cost stated
1A1ft: CAO (Penalise reversed states here)

(c) Average expenditure $\frac{37+38+34+21}{4}=\frac{130}{4}=$ £32 500 M1 A1 2

Notes

1M1: Sum of four arcs /4 (do not isw here if they 'add' to this method)
1A1: CAO (32 500 gets both marks)

Special cases (and misreads)

SC1 Maximin: treat as misread. **MAX 11/13**

SC2 Maximum: 1M1,1A1; 2M0; 3M1,4A1ft,5A0,6A1ft,
M1A1ft M1A1ft **MAX 9/13**

SC3 Minimum: Marks awarded as above SC2

SC4 Maximax: 1M1,1A1; 2M0; 3M1,4A0,5A0,6A0,
M1A1ft M1A1ft **MAX 7/13**

SC5 Minimin: Marks awarded as above SC4

SC6 Working forwards: 1M1,1A0; 2M0; 3M1,4A0,5A0,6A0,
M1A1ft M1A1ft **MAX6/13**

SC1 Maximim

Stage	State	Action	Dest.	Value
	G	GT	T	17 *
1	H	HT	T	21 *
	I	IT	T	29 *
2	D	DG	G	min(22, 17) = 17
		DH	H	min(31, 21) = 21
	E	EH	H	min(39, 29) = 29 *
		EI	I	min(39, 29) = 29 *
	F	FI	I	min(52, 29) = 29 *
3	A	AD	D	min(41, 22) = 21
		AE	E	min(38, 29) = 29 *
	B	BE	E	min(44, 29) = 29 *
	C	CE	E	min(36, 29) = 29 *
		CF	F	min(35, 29) = 29 *
4	S	SA	A	min(37, 29) = 29 *
		SB	B	min(39, 29) = 29 *
		SC	C	min(41, 29) = 29 *

SC2 Maximum route

Stage	State	Action	Dest.	Value
	G	GT	T	17 *
1	H	HT	T	21 *
	I	IT	T	29 *
2	D	DG	G	22 + 17 = 39
		DH	H	31 + 21 = 52 *
	E	EH	H	31 + 21 = 52 *
		EI	I	39 + 29 = 68 *
	F	FI	I	52 + 29 = 81 *
3	A	AD	D	41 + 52 = 93
		AE	E	38 + 68 = 106 *
	B	BE	E	44 + 68 = 112 *
	C	CE	E	36 + 68 = 104
		CF	F	35 + 81 = 116 *
4	S	SA	A	37 + 106 = 143
		SB	B	39 + 112 = 151
		SC	C	41 + 116 = 157 *

Route: SCFIT

SC3 Minimum route

Stage	State	Action	Dest.	Value
	G	GT	T	17 *
1	H	HT	T	21 *
	I	IT	T	29 *
2	D	DG	G	22 + 17 = 39 *
		DH	H	31 + 21 = 52
	E	EH	H	34 + 21 = 55 *
		EI	I	39 + 29 = 68

	F	FI	I	52 + 29 = 81 *
3	A	AD	D	41 + 39 = 80 *
		AE	E	38 + 55 = 93
	B	BE	E	44 + 55 = 99 *
	C	CE	E	36 + 55 = 91 *
		CF	F	35 + 81 = 116
4	S	SA	A	37 + 80 = 117 *
		SB	B	39 + 99 = 138
		SC	C	41 + 91 = 132

Route: SADGT

SC4 Maximax route

Stage	State	Action	Dest.	Value
	G	GT	T	17 *
1	H	HT	T	21 *
	I	IT	T	29 *
2	D	DG	G	max(22, 17) = 22
		DH	H	max(31, 21) = 31 *
	E	EH	H	max(34, 21) = 34
		EI	I	max(39, 29) = 39 *
	F	FI	I	max(52, 29) = 52 *
3	A	AD	D	max(41, 31) = 41
		AE	E	max(38, 29) = 39 *
	B	BE	E	max(44, 29) = 44 *
	C	CE	E	max(36, 29) = 39
		CF	F	max(35, 52) = 52 *
4	S	SA	A	max(37, 29) = 39
		SB	B	max(39, 44) = 44
		SC	C	max(41, 52) = 52 *

Route: SCFIT

SC1 Minimim

Stage	State	Action	Dest.	Value
	G	GT	T	17 *
1	H	HT	T	21 *
	I	IT	T	29 *
2	D	DG	G	min(22, 17) = 17 *
		DH	H	min(31, 21) = 21
	E	EH	H	min(34, 21) = 21 *
		EI	I	min(39, 29) = 29
	F	FI	I	min(52, 29) = 29 *
3	A	AD	D	min(41, 17) = 17 *
		AE	E	min(38, 21) = 21
	B	BE	E	min(44, 21) = 21 *
	C	CE	E	min(36, 29) = 21 *
		CF	F	min(35, 29) = 29
4	S	SA	A	min(37, 17) = 17 *
		SB	B	min(39, 21) = 21

		SC	C	min(41, 21) = 21

Route: SADGT

SC6 Working towards S to T

Stage	State	Action	Dest.	Value
	A	AS	S	37 *
1	B	BS	S	39 *
	C	CS	S	41 *
2	D	DA	A	max(41, 37) = 41 *
	E	EA	A	max(38, 37) = 38 *
		EB	B	max(44, 39) = 44
	F	FC	C	max(36, 41) = 41
3	G	GD	D	max(22, 41) = 41 *
	H	HD	D	max(31, 41) = 41
		HE	E	max(34, 38) = 38 *
	I	IE	E	max(39, 38) = 39 *
		IF	F	max(52, 41) = 52
4	T	TG	G	max(17, 41) = 41
		TH	H	max(21, 38) = 38 *
		TI	I	max(29, 39) = 39

Route: SAEHT

[13]

3. (a)

Stage	**State (in £1000s)**	**Action (in £1000s)**	**Dest. (in £1000s)**	**Value (in £1000s)**	
	250	*250*	*0*	*300**	
1	*200*	*200*	*0*	*240**	
	150	*150*	*0*	*180**	
	100	*100*	*0*	*120**	
	50	*50*	*0*	*60**	
	0	*0*	*0*	*0**	
	250	*280*	*0*	*200 + 0 = 280*	
		200	*50*	*235 + 60 = 295*	
		150	*100*	190 + 120 = 310*	
		100	*150*	125 + 180 = 305	1M1A1
		50	*200*	65 + 240 = 305	
		0	*250*	0 + 300 = 300	
2	*200*	*200*	*0*	235 + 0 = 235	
		150	*50*	190 + 60 = 250*	
		100	100	125 + 120 = 245	A1
		50	150	65 + 180 = 245	
		0	200	0 + 240 = 240	
	150	*150*	0	190 + 0 = 190*	2M1
		100	50	125 + 60 = 185	
		50	100	65 + 120 = 185	A1
		0	150	0 + 180 = 180	
	100	100	0	125 + 0 = 125*	
		50	50	65 + 60 = 125*	A1

		0	100	0 + 120 = 120	
	50	50	0	65 + 0 = 65*	
		0	50	0 + 60 = 60	
	0	0	0	0 + 0 = 0*	3M1
3	250	250	0	300 + 0 = 300	A1ft
		200	50	230 + 65 = 295	
		150	100	170 + 125 = 295	
		100	150	110 + 190 = 300	
		50	200	55 + 250 = 305	
		0	250	0 + 310 = 310*	

Maximum income £310 000 B1

Scheme	1	2	3
Invest (in £1000s)	100	150	0

B1 10

(b) Stage: Scheme being considered B1

State: Money available to invest B1

Action: Amount chosen to invest B1 3

[13]

4. (a) Maximin : we seek a route where the shortest arc used is a great as possible.
Minimax : we seek a route where the longest arc used is a small as possible. B2,1,0 2

(b)

Stage	State	Action	Dest.	Value	
	G	GR	R	132*	
1	H	HR	R	175*	M1A1
	I	IR	R	139*	
	D	DG	G	min (175,132) = 132	M1A1
		DH	H	min (160,175) = 160*	
2	E	EG	G	min (162,132) = 132	
		EH	H	min (144,175) = 144*	A1
		EI	I	min (102,139) = 102	
	F	FH	H	min (145,175) = 145*	
		FI	I	min (210,139) = 139	
	A	AD	D	min (185,160) = 160*	
		AE	E	min (279,144) = 144	M1A1ft
3	B	BD	D	min (119,160) = 119	
		BE	E	min (250,144) = 144*	A1ft
		BF	F	min (123,145) = 123	
	C	CE	E	min (240,144) = 144	
		CF	F	min (170,145) = 145*	
	L	LA	A	min (155,160) = 155*	A1ft
4		LB	B	min (190,144) = 144	
		LC	C	min (148,145) = 145	

Maximin route: LADHR A1ft 5

[12]

5. (a)

Stage	State	Action	Destination	Value	
	J	JY	Y	98*	
1	K	KY	Y	94*	B1
	L	LY	Y	86*	
	G	GJ	J	max(79, 98) = 98*	M1
		GK	K	max(98, 94) = 98*	
2	H	HK	K	max(95, 94) = 95	A1A1
		HL	L	max(72, 86) = 86*	
	I	IL	L	max(56, 86) = 86*	
	C	CG	G	max(50, 98) = 98*	
	D	DG	G	max(92, 98) = 98	M1
3		DH	H	max((81, 86) = 86*	A1A1ft
	E	EH	H	max(89, 86) = 89*	
	F	FH	H	max(84, 86) = 86*	
		FI	I	max(72, 86) = 86*	
	A	AC	C	max(95, 98) = 98	M1
		AD	D	max(86, 86) = 86*	A1ft
4		AE	E	max(63, 89) = 89	
	B	BE	E	max(88, 89) = 89	
		BF	F	max(87, 86) = 87*	
5	X	XA	A	max(55, 86) = 86*	A1ft
		XB	B	max(85, 87) = 87	

X A D H L Y (minimax = 86) M1A1ft 12

(b) X B F $\begin{matrix} \diagup H \diagdown \\ \diagdown I \diagup \end{matrix}$ L Y (minimax = 87) one M1A1 2

[14]

6. (a) Any part of an optimal path is itself optimal B1

(b) The route chosen such that the maximum arc length is as small as possible B1

(c) e.g. Maximising freight by minimising fuel needed when planning multiple stage light aircraft journey B2, 1, 0

B1 cao ("port", "section", OK; "arc", "stage", activity", "event", not)
B1 cao (not min of max rate, not minimize largest arc)
B2 cao
B1 cloze "Bod" gets B1

[4]

7.

Stage	State	Action	Value		
1	H	HT	4*		
	I	IT	3*		
	J	JT	12*		
	K	KT	20*	M1 A1	2
2	D	DH	2 +4 = 6		
		DI	4 + 3 = 7*	M1 A1	
	E	EH	3 + 4 = 7*		
		EI	4 + 3 = 7*		
	F	FJ	10 + 12 = 22*		
		FK	–8 + 20 = 12		
	G	GJ	10 + 12 = 22		
		GK	17 + 20 = 37*	A1	3
3	A	AD	3 + 7 = 10		
		AE	2 + 7 = 9	M1 A1ft	
		AF	–5 + 22 = 17*		
	B	BD	3 + 7 = 10		
		BE	2 + 7 = 9		
		BF	–6 + 22 = 16*		
	C	CF	8 + 22 = 30*		
		CG	–15 + 37 = 22	A1 ft	3
4	S	SA	2 + 17 = 19		
		SB	3 + 16 = 19	M1 A1ft	2
		SC	–10 + 30 = 20*		

Route S C F J T £20 000 M1 A1 2

[12]

8. e.g.

Stage	State	Action	Dest	Value
1 (Sept)	2	2	0	200 + 200 = 400 *
	1	3	0	200 + 100 = 300 *
	0	4	0	200 = 200 *
2 (Aug)	2	5	2	200 + 200 + 500 + 400 = 1300
		4	1	200 + 200 + 300 = 700
		3	0	200 + 200 + 200 = 600 *
	1	5	1	200 + 100 + 500 + 300 = 1100
		4	0	200 + 100 + 200 = 500 *
	0	5	0	200 + 500 + 200 = 900 *
3 (Jul)	2	5	0	200 + 200 + 500 + 900 = 1800 *
4 (Jun)	2	3	2	200 + 200 + 1800 = 2200 *
	1	4	2	200 + 100 + 1800 = 2100 *
	0	5	2	200 + 500 + 1800 = 2500 *
5 (May)	0	5	2	200 + 500 + 2200 = 2900
	1	4	2	200 + 2100 = 2300 *
	0	5	2	200 + 2500 = 2700 *

Month	May	June	July	August	September	M1 A1ft
production schedule	4	4	5	5	4	

Cost £2300 A1ft 3

[12]

9. (a) The route from start to finish in which the arc of minimum length is as large as possible. B2, 1, 0
e.g. must be pratical, involve choice of route, have are 'cuts'. B1 3

(b)

Stage	State	Action	Value		
1	H	HK	18(*)	M1 A1	2
	I	IK	19(*)		
	J	JK	21(*)		
2	F	FH	min (16,18) = 16		
		FI	min (23,19) = 19(*)	M1 A1 A1	3
		FJ	min (17,21) = 17		
	G	GH	min (20,18) = 18		
		GI	min (15,19) = 15		
		GJ	min (28,21) = 21(*)		
3	B	BG	min (18,21) = 18(*)		
	C	CF	min (25,19) = 19(*)	M1 A1ft	
		CG	min (16,21) = 16		
	D	DF	min (22,19) = 19(*)		
		DG	min (19,21) = 19(*)		
	E	EF	min(14,19) = 14(*)		
4	A	AB	min (24,18) = 18	A1ft	3
		AC	min (25,19) = 19(*)		
		AD	min (27,19) = 19(*)		
		AE	min (23,14) = 14		

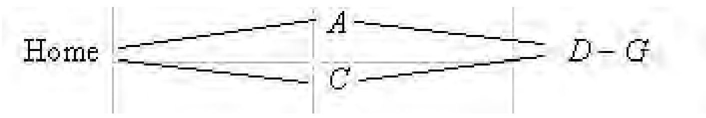
(c) Routes *A C F I K*, *A D F I K*, *A D G J K* A1ft A1ft A1ft 3

[14]

10. (a) Stage – Number of weeks to finish B1
State – Show being attended B1
Action – Next journey to undertake B1 3

(b) <u>eg</u>

Stage	State	Action	Value		
1	*F* *G* *H*	*F* – Home *G* – Home *H* – Home	500 – 80 = 420 * 700 – 90 = 610 * 600 – 70 = 530 *	M1 A1	
2	*D*	*DF* *DG* *DH*	1500 – 200 + 420 = 1720 1500 – 160 + 610 = 1950 * 1500 – 120 + 530 = 1910	M1 A1ft A1 ft	
	E	*EF* *EG* *EH*	1300 – 170 + 420 = 1550 1300 – 100 + 610 = 1810 * 1300 – 110 + 530 = 1720	A1	
3	A	*AD* *AE*	900 – 180 + 1950 = 2670 * 900 – 150 + 1810 = 2560	M1 A1 ft	
	B	*BD* *BE*	800 – 140 + 1950 = 2610 * 800 – 120 + 1810 = 2490	A1 ft	
	C	*CD* *CE*	1000 – 200 + 1950 = 2750 * 1000 – 210 + 1810 = 2600	A1	
4	Home	Home – *A* Home – *B* Home – *C*	–70 + 2670 = 2600 * –80 + 2610 = 2530 –150 + 2750 = 2600 *	M1 A1	12

(c) 

B2 ft 1 ft 0

Total profit £2 600 B1 ft 3

[18]

11. (a) Total cost = 2 × 40 + 350 + 200 = £630 M1 A1 2

(b)

Stage	Demand	State	Action	Destination	Value		
(2) Oct	(5)	(1)	(4)	(0)	(590 + 200 = 790)		
		(2)	(3) (4)	(0) (1)	280 + 200 = 480 630 + 240 = 870	M1 A1	
		(3)	(2) 3 4	0 1 2	320 + 200 = 520 320 + 240 = 560 670 + 80 = 750	M1 A1	4
3 Sept	3	0	4	1	550 + 790 = 1340	M1 A1	
		1	3 4	1 2	240 + 790 = 1030 590 + 480 = 1070	M1 A1 ft	
4 Aug	3	0	3 4	0 1	200 + 1340 = 1540 550 + 1030 = 1580	M1 A1 ft	6

Month	August	September	October	November
Make	3	4	4	2

M1 A1

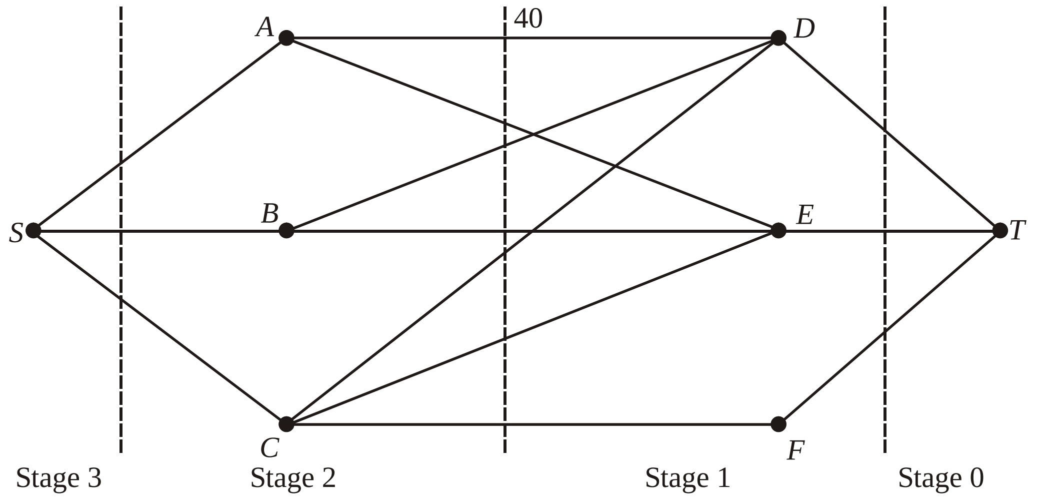
cost = £1540 A1 ft 3

(c) Profit per cycle = 13 × 1400 Cost of Kim's time = £2000
= 18 200 Cost of production = £1540 B1

∴ Total profit = 18 200 – 3540 M1
= <u>£14 660</u> A1 ft 3

[18]

12.



The states are the vertices.

Stage	Initial state	Action	Final state	Value
1	*D*	*DT*	*T*	30
	E	*ET*	*T*	40
	F	*FT*	*T*	20
2	*A*	*AD*	*D*	max (40, 30) = 40*
		AE	*E*	max (55, 40) = 55
	B	*BD*	*D*	max (50, 30) = 50*
		BE	*E*	max (60, 40) = 60
	C	*CD*	*D*	max (45, 30) = 45
		CE	*E*	max (50, 40) = 50
		CF	*F*	max (30, 20) = 30*
3	*S*	*SA*	*A*	max (40, 20) = 40*
		SB	*B*	max (50, 30) = 50
		SC	*C*	max (40, 30) = 40*

Stage 1 M1 A1

Stage 2 M1 A1 A1 A1

Stage 3 (*) M1 A1

Tracing back there are two routes

SC, *CF*, *ft*. ⇒ *SC FT*

SA, *AD*, *DT*. ⇒ *SADT* M1 A1

Max altitude on these routes is 40 (× 100 ft) = 4000 ft. A1

[13]

13.

60	56	50
50	45	35
60	50	45

Subtract each element from 100 (or any number ≥ 60)

40	44	50
50	55	65
40	50	55

(obtaining new table) M1 A1

Row minima are 40, 50, 40

Reducing rows:

0	4	10
0	5	15
0	10	15

(row reduction) M1 A1

Column minima are 0, 4, 10

Reducing columns

0	0	0
0	1	5
0	6	5

(row reduction) M1 A1

Zeroes can be covered by two lines: thus no allocation possible.

Smallest uncovered number is 1, so we obtain

1	0	0
0	0	4
0	5	4

(new table) M1 A1

Three lines are now required to cover zeroes. So allocation can be made:		B1
Must chose (1,3)	$(D \to 3)$	
$\Rightarrow$ (2, 2) must be chosen	$(S \to 2)$	M1 A1
$\Rightarrow$ (3, 1) must be chosen	$(B \to 1)$	
Returning to original matrix, maximum profit is 50 + 45 + 60 = £155,000		A1

[13]

1. No Report available for this question.

2. This question was a good source of marks for many. The vast majority completed the table correctly, but a significant number answered this as a minimum route problem, others as a maximum route problem and a few as a maximin problem. A very few worked forwards or reversed the states. Most candidates selected a correct route, and value, for the problem they solved.

Part (c) proved a challenge for many, with a wide range of incorrect methods seen.

3. This was a challenging question, requiring candidates to think carefully, the examiners were pleased by the way in which many of the candidates tackled this question. Most candidates were able to make good progress with part (a), with only a small minority unable to attempt anything or just the first four rows. Most worked correctly through the subsequent stages without much difficulty; although some omitted stage 2 state 50 and/or 0 and some did unnecessary work in stage 3. Some poor arithmetic was seen with 125 + 60 = 195 and 190 + 60 = 240 being fairly frequently seen. Those who completed the table were then able to answer the subsequent questions, although some were unable to state the correct maximum income. In part (b) most were able to define ‘scheme’ correctly, but the definitions of state and action challenged many.

4. This was often a rich source of marks for most. It was rare to see candidates that gained both marks in part (a), with poor use of terminology seen, especially route/arc/path/ but also imprecise use of vertex, distance and value. Part (b) was often well done with many fully correct solutions seen. All the ‘usual’ misreads were also seen with candidates finding the minimax, minimum or maximum routes. A much smaller number of candidates than usual reversed the states or worked forwards, and centres should be congratulated in this.

5. This question was generally done well, with only minor numerical slips in some candidates working, or the omission of some stars (particularly GH and FH). A number of candidates incorrectly calculated the minimum route instead of the minimax. Only a few candidates attempted the other varieties (maximum, maximax or minimin) or tried to answer the problem by working forwards. As always a small number of candidates did not choose their states sensibly, and consequently had difficulty carrying the data found in earlier stages to the later ones. Most candidates successfully gave both routes.

6. This question was often poorly answered. Many candidates did not seem t know Bellman’s principle and even those that did were often unable to give an accurate statement. Many candidates gave a confused definition of a minimax route, often including statements such as “minimise the maximum route”. In the final part, candidates often had some idea of a practical problem, such as “walkers in mountains” or “a plane making a multistage journey” but then sometimes failed to give a full statement.

7. This question was well done by the vast majority of candidates, although some lost marks to minor arithmetical errors or failing to indicate their maximum values. Some candidates lost marks through the omission of one column such as “State” or more seriously by confusing the order of actions within one stage. A few candidates chose to do something other than maximise and some worked forwards rather than backwards. Other minor errors occurred in stating the route and sometimes omitting the units of the profit.

8. This was poorly attempted by many candidates. Too many started at May rather than September and many forgot to include earlier values in later calculations. Some did not read the questions carefully and tried to make more than 5 or store more than 2 in any one month.

9. Many candidates did not present a good definition of a maximin route and were not able to offer a good practical example. Part (b) was often well-answered although some candidates found the minimax or minimum or maximum route instead. Other common slips were a few numerical errors, failure to indicate the optimal choice at each state and swapping the order of the states, making it impossible to follow the route back though the table.

10. This question was the most variable in terms of quality of response and method of answering. Many candidates did not give correct definitions in part (a). The majority of candidates then completed the table with a high degree of success. The most common errors were: omitting the first or the last stage from the table, reversing actions, interchanging states, arithmetic slips, working forwards and attempting to minimise costs and then subtracting these from the profit. Candidates who completed the table were then usually able to complete part (c) correctly.

11. This question was either very well answered, with candidates scoring full (or nearly full) marks, or quite poorly answered. A surprising number of candidates miscalculated the answer to part (a). Most candidates were able to gain some marks in part (b), and some full marks. Some candidates simply wrote out the order book as their production schedule. Part (c) caused a few problems with one factor often omitted – surprisingly the most commonly omitted one was the production cost from part (b).

12. No Report available for this question.

13. No Report available for this question.